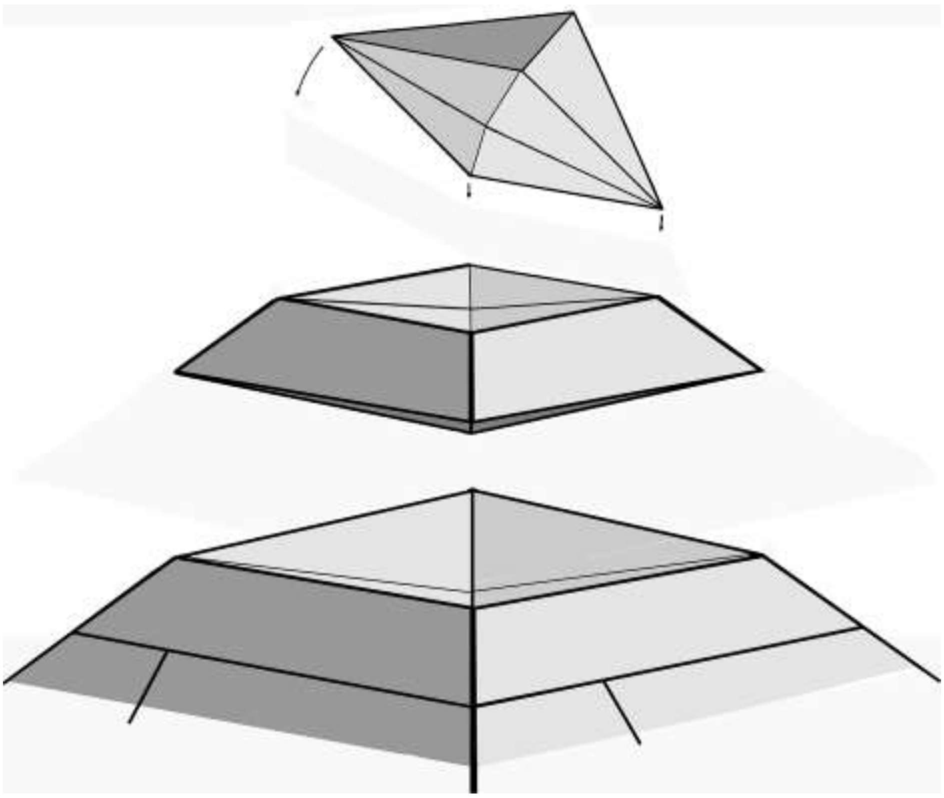
THE REVISED AND COMPLETE ARTICLE ON

THE PYRAMIDION OF THE SATELLITE PYRAMID OF KHUFU, G1D

By Josef Dorner

A RECTIFICATION OF THE APPENDIX TO Z. HAWASS, THE DISCOVERY OF THE PYRAMIDION OF THE SATELLITE PYRAMID OF KHUFU, *IUBILATE CONLEGAE, STUDIES IN MEMORY OF ABDEL AZIZ SADEK,*

***VARIA AEGYPTIACA, VOL 11, SAN ANTONIO, 1998,* 105 - 124.**



Drawn By Nabil Swelim: April 1996

The top of this pyramidion is missing and a great part of the faces damaged. The horizont edges are broken away all along the base. Of the sloping edges only two are partly preserved, in the southeast 25 cm and in the southwest 50 cm. (These directions refer only to the present position).

The lower side, only slightly damaged, is not flat but divided along the diagonals in four plains, each of them sloping from one of the horizontal edges towards the centre. This protruding part was inserted into the second stone course and remained therefore rough.

Since neither the top nor one of the corners are preserved and the base is completely destroyed, we had to choose an indirect method to obtain measurements: A large aluminium ruler was placed with its flat side, against the lower plain and gradually shifted until its edge was in line with the upper face. Thus on each side the line of the original base was reconstructed and in 130 mm distance a parallel was drawn on the preserved part of the face (fig. 1).

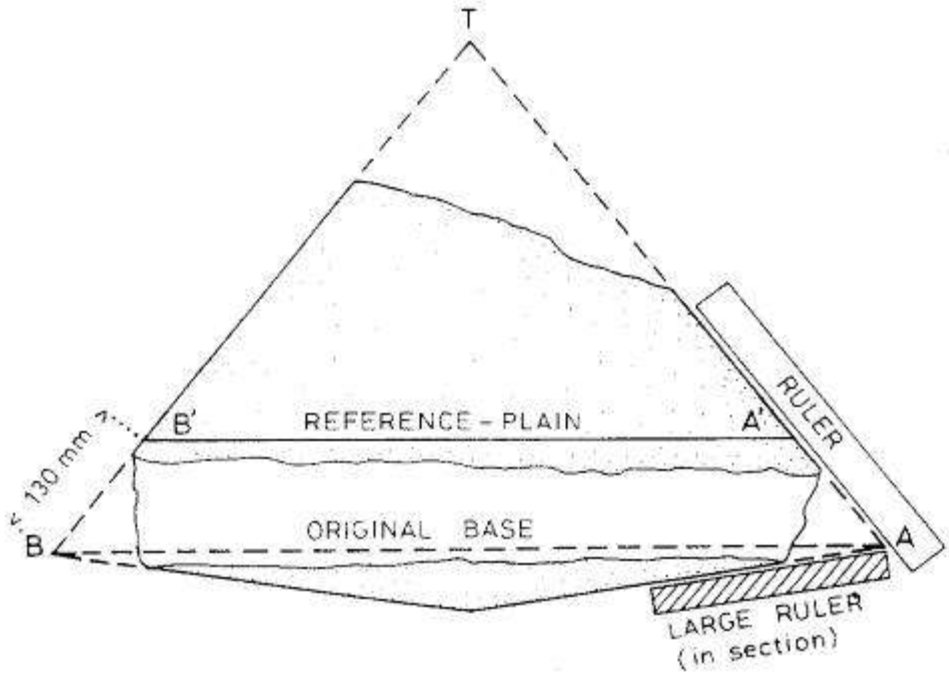
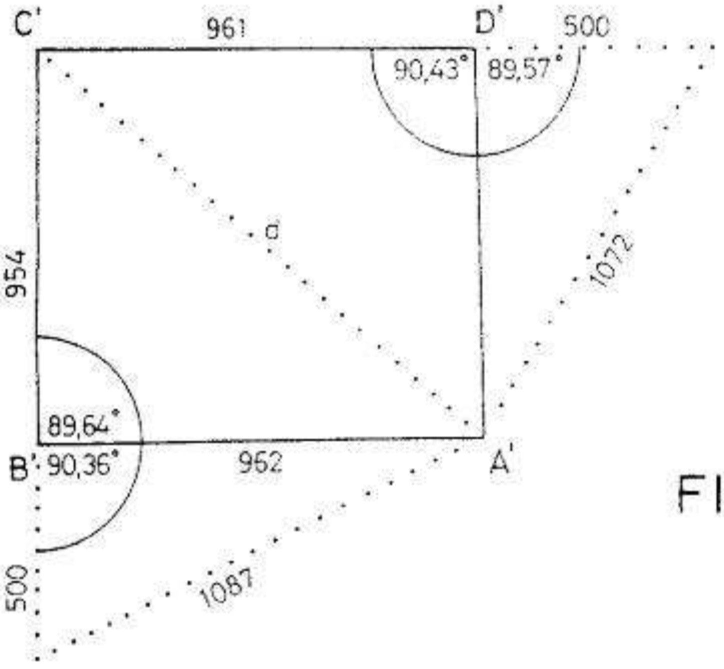


FIG. 1

These four parallels with points A', B', C', D' are the base of a smaller pyramid, which is similar to the pyramidion and can be measured. The position of the missing top and of the destroyed corners C´ and D´ had to be found by extending the edges. This caused some inaccuracy in the lengths, approximately 2 mm for the horizontal and 4 mm for the sloping ones.

From the base only the four sides could be measured but not the diagonals. In order to gain the fifth element necessary to define its shape and as a control, the angles in B´and D´ were determined by auxiliary triangles. The distances measured (in mm) and the angles derived from them are given in fig. 2 below.



With this data the diagonal A´C´ can be calculated twice:

A´B´C´: d = 1351
A´C´D´: d = 1357
mean : d = 1354

FIG. 2

Taking the means as being close to the truth the figure is defined and we receive for the adjusted angles:

$\alpha_{A'}$ = 90,01°; $\alpha_{B'}$ = 89,93°; $\alpha_{C'}$ = 89,95°; $\alpha_{D'}$ = 90,11°

The base has almost exactly a rectangular shape . In neglecting the little difference in the length of the corresponding sides we set:

A´B´ = C´D´ = 962 and B´C´ = D´A´ = 953

The sloping edges in the North (C´T and D´T) are completely destroyed and those in the South only partly preserved. Extending them until they intersect (T), we found:

A´T = 911 and B´T = 916

On the western face the angle included by the base and the southwestern edge was derived by an auxiliary triangle ($\alpha_{B'}$ = 58,13°) and from it the length of the sloping edge in the Northwest derived: C´T = 909.

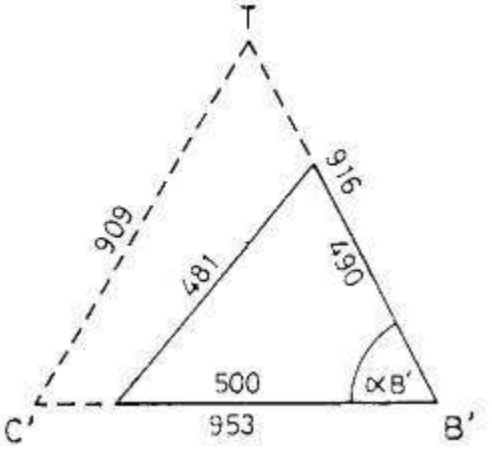


FIG. 3

By this data the pyramid above the reference plain is defined and we are able to calculate all its elements (comp. fig. 4):

in triangle A´B´T			in triangle B´C´T		
P´T	=	777	Q´T	=	778
P´A´	=	476	Q´B´= P´M´	=	484
P´B´= Q´M´	=	486	Q´C	=	469

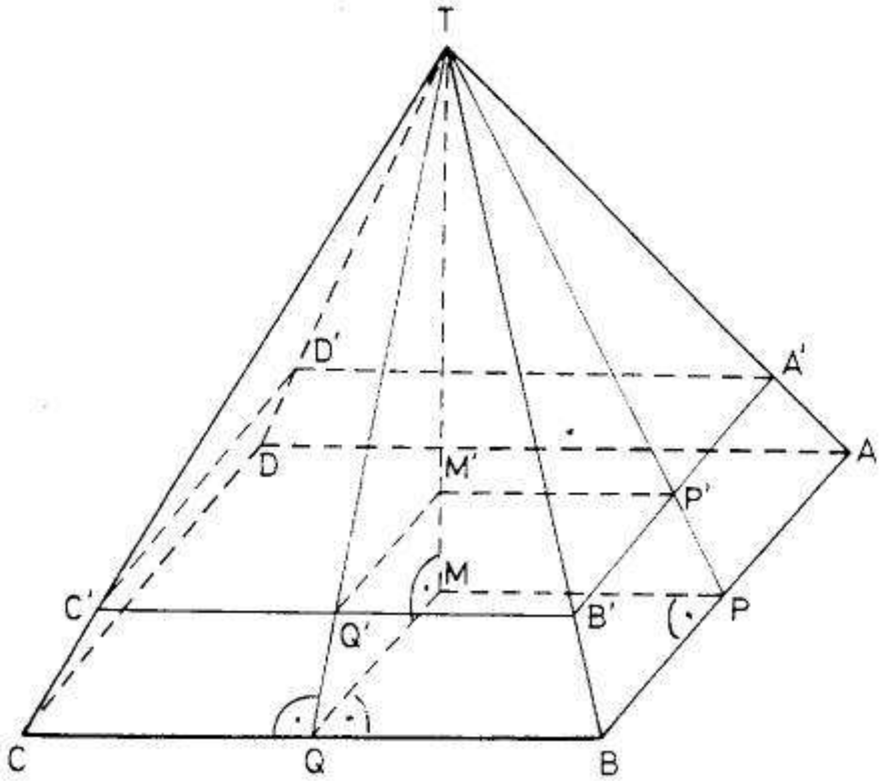


FIG. 4

the height:

$$M'T = \sqrt{A'T^2 - A'P'^2 - B'Q'^2} = 607$$

$$M'T = \sqrt{B'T^2 - B'P'^2 - B'Q'^2} = 607$$

the sloping edge in the Northeast:

$$D'T = \sqrt{M'T^2 + C'Q'^2 + A'P'^2} = 903$$

Enlarging with the factor k the corresponding elements of the pyramidion are found:

$$k = \frac{TP}{TP'} = \frac{TP' + 130}{TP'} = 1,167$$

$$k = \frac{TQ}{TQ'} = \frac{TQ' + 130}{TQ'} = 1,167$$

The shape of the pyramidion

height. MT = 709 mm (hight above bais plain)

base:

S:	AB	=	1123 mm
W:	BC	=	1112 mm
N:	CD	=	1123 mm
E:	DA	=	1112 mm
mean:		=	1118 mm

sloping edges:

SE:	AT	=	1063 mm
SW:	BT	=	1069 mm
NW:	CT	=	1060 mm
NE:	DT	=	1054 mm

horizontal distances from M to basis.

S:	PM	=	565 mm
W:	QM	=	567 mm
N:	BC-PM	=	547 mm
E:	AB-QM	=	556 mm
mean:		=	559 mm

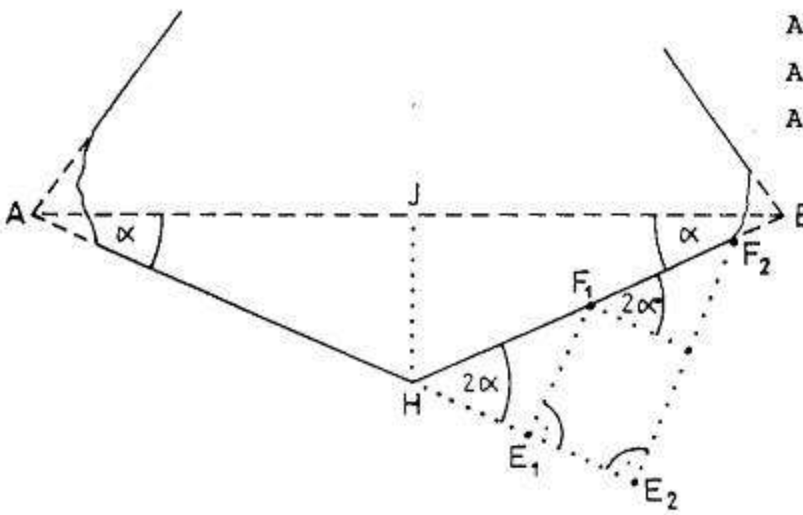
slope:

S:	51,45°	=	51°27'
W:	51,35°	=	51°21'
N:	52,35°	=	52°21'
E:	51,90°	=	51°53'
α mean:	51,76°	=	51°45'

The top of the pyramidion is not exactly above the centre of the base but shifted 11 mm towards the corner D. The faces therefore differ considerably in their slope.

The lower part

The underside of the pyramidion is not flat but formed as a low and inverted pyramid, its base-lines are common with the upper one. In order to obtain its height we measured in the vertical plain disecting it from east to west the following distances (fig. 5):



$AE_1 = 800;\ E_1E_1 = 58$
$AE_2 = 1000;\ E_2E_2 = 110$
$AB = 1123$

FIG. 5

Deriving from the triangle F_1F_2K:

$$tg\ 2\alpha = \frac{E_2F_2 - E_1F_1}{AE_2 - AE_1} \quad ; \quad \alpha = 7{,}3°$$

we find the height:

$$JH = \frac{AB \cdot tg\,\alpha}{2} = 72\ mm$$

This height is 3 mm less than 4 fingers, which might have been intended.

A block from the third course

Some interesting observations could also be obtained from a large block, which represents little more than half of the entire third stone layer of the pyramid (counting from the top). Its three faces are partly destroyed especially at the sloping edges and along the base. The upper side, better preserved, has been hollowed out in order to receive the protruding underside of the second course, which had a shape as described for the pyramidion. The underside of the block is flat.

In D, 320 mm distance from corner B, the last preserved point of this edge (comp. fig. 6) we placed a long ruler parallel to AB and measured from it the vertical distances to E (14 mm) and F (15 mm). Enlarging the mean by 912 : 320 we obtain the depth of the point M below the horizontal edges with 41 mm. The slope of the triangle ABM is 2,6°, much less than the slope observed at the protruding underside of the pyramidion.

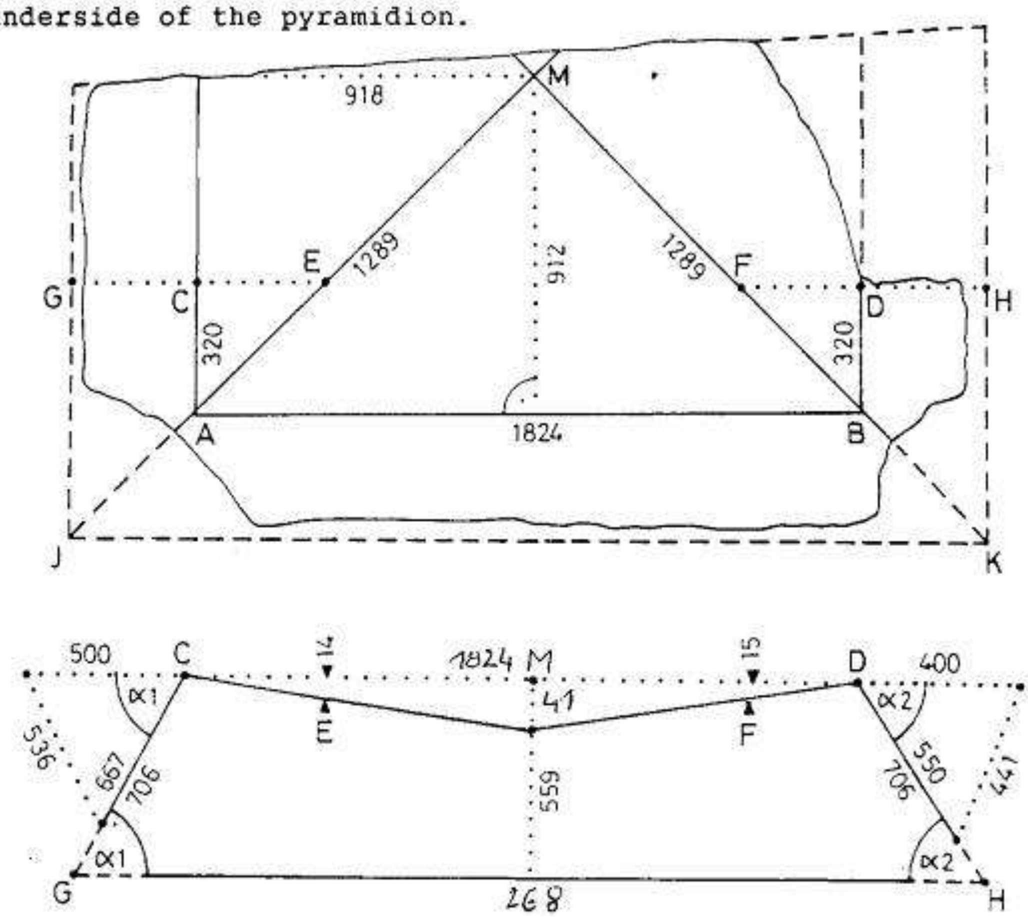


FIG. 6

In the same section the slope of the faces was derived from two auxiliary triangles (all data in mm given in fig. 7): α_1 = 52,33° α_2 = 52,47°, mean: α = 52,40°.

Assuming an error of ± 2 mm in the sides the expected error in the angles is about ± 0,3°. The difference between α_1 and α_2 is therefore not significant.

The height of the block was measured three times:

near G:	559 mm
between J and K:	560
near H:	558
mean:	559

Extending the sloping face and the underside of the block with two rulers we found four points of the original base lines and measured their distances from the upper edges. The results vary considerably, between 702 and 709 with a mean of 706 mm, because the faces are not plain and the underside is rough.

The slope (52,40°) and the height (559) give the recess of the faces (430) and adding the latter twice to the length of the upper edge (1824) we receive the length of the base: GH = JK = 2685 mm.

Reconstruction

The possibility of determining accurately the original dimensions of these two blocks is restricted, because both are heavily damaged. Only a few lengths, which could be measured directly, are accurate to ± 2 mm, the others are uncertain to 3 or 4 mm. It is therefore likely that the mason´s work was more accurate than it appears from our results.

The mean-slope of the pyramidion coincides almost exactly with that of the king´s and the three queen´s pyramids in the funerary complex of Cheops (51°50´)[1] and there is no doubt that the same slope was employed also for the new satelite pyramid. This slope is equal with a pattern of 28 : 22, a *seked* of 5 palms, 2 digits and a proportion between height and base of 7 : 11.

In the search of the intended size of the pyramidion two possibilities must be considered:
a base of 60 D (1125)[2] with a height of $38^{2}/_{11}$ D (716)
or a height of 38 D (713) with a base of 59 $^{5}/_{7}$D(1120).

The second pair differs only little from our results and is therefore much more likely.

From the third course downwards the casing blocks had flat, horizontal bases. The three uppermost layers, however, were linked by a peculiar construction and represent a special unit. Its base with a length of 143 D (2681) was certainly chosen deliberately, because it corresponds with a height of exactly 91 D (1706). Such pairs, both elements with integer numbers exist only in levels 7 D distant from each other.

For the height of the third course we received 3 mm less than 30 D (562). The difference is probably due to the missing edges of the base, on which the block was resting. We observed that the faces are not excactly plain, especially the section C-D which is distinct convex. Along the upper edges the masons did not edge away enough from the block. The preserved edge is 27 mm longer than $95^{6}/_{7}$ D (1797) and the faces therefore more than 0,5° too steep.
Assuming that the third course rested 91 D below the top in the height, which corresponds with the length of its base (143 F), the second course would have been 23 D (43) high and the slope of its faces only 50°, due to the faulty length of AB. In fig. 7 the deviations from the intended shape are shown in scale 1 : 1.

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[1] V. Maragioglio a. C. Rinaldi, *L´Architettura delle Piramidi Menfite IV*. Rapollo 1965, pp. 18, 80, 86, 92.

[2] 1 D = 18,75 mm; corresponding with a cubit of 525 mm.

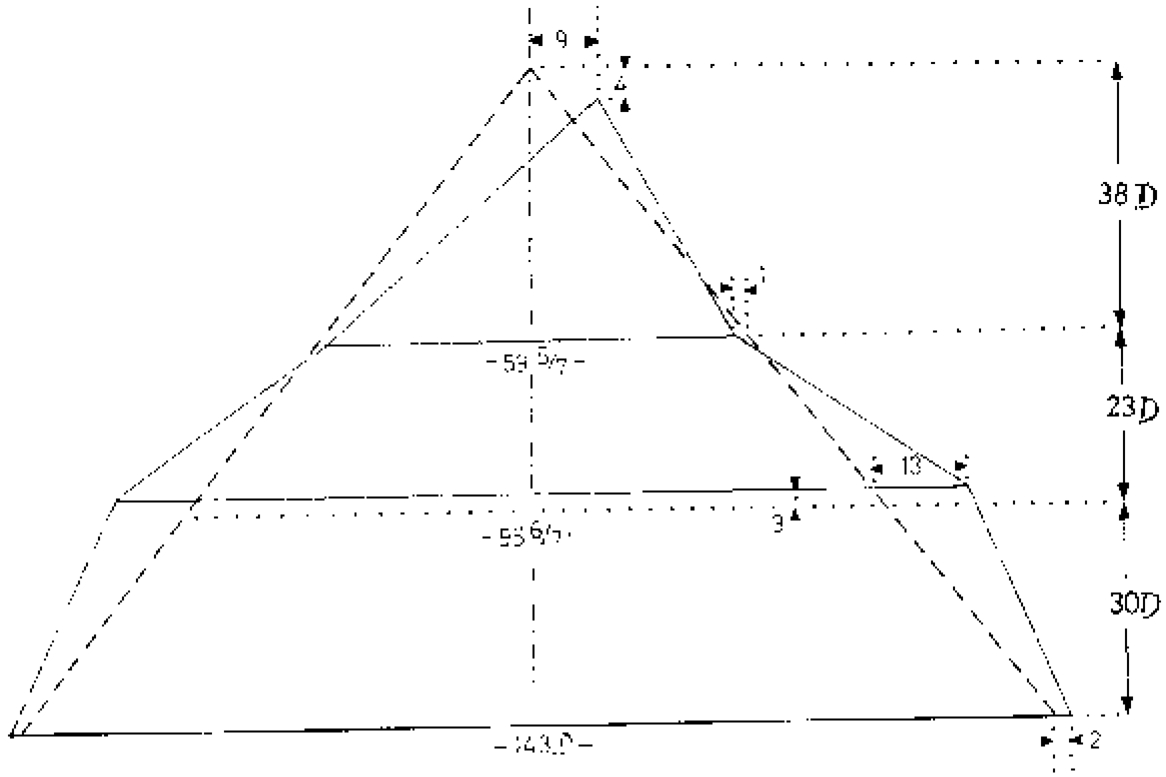

FIG. 7

I am very grateful to Dr. Zahi Hawass for his kind invitation to take the measurements of these blocks. I must also thank Madam Nevin Mohamed Mustafa and Dr. Peter Janosi for their support of this work.